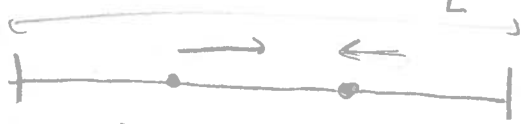
Crash course in Statistical Mechanics.

isolated system / the whole universe

E conserved (micro canonical)

how big is the part of phase space where you can go?

2 particles on a line in 1 dimension (ideal gas)

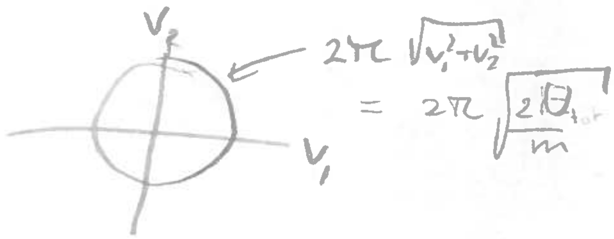


1d box

particle 1: real space: L

2 " " " L

total $E_{kin} = (v_1^2 + v_2^2)\frac{1}{2}m$



volume of the phase space is

$2\pi L^2 \sqrt{\frac{2E}{m}}$

If we had N particles:

vol $\propto L^N \left(\frac{2U}{m}\right)^{\frac{N-1}{2}}$

in 3d:

vol $\propto V^N \left(\frac{2U}{m}\right)^{\frac{3N-1}{2}}$

This volume gives the entropy

$S = k \ln(\text{vol})$

ideal gas: $S = Nk\left\{\ln\left[\frac{V}{N}\left(\frac{4\pi m U}{3N h^2}\right)^{3/2}\right] + \frac{5}{2}\right\}$

↑ Planck's constant.

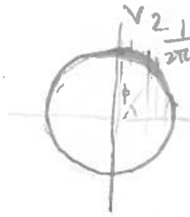
not isolated system (part of a bigger system)

many particles

we are interested in this one, the rest is heat bath

What is the total energy of this particle? Not conserved.

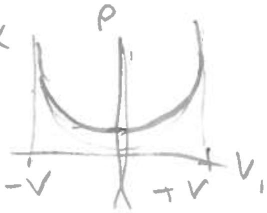
Back to two particles in 2d



$\frac{1}{2\pi v} v \, d\phi \rightarrow \frac{d\phi}{dx} dX \rightarrow \frac{1}{\sin\phi} dx \times \frac{1}{2\pi v} \frac{1}{\sqrt{1-\frac{v_1^2}{v^2}}} dx$

$x = v \cos\phi$

$\longrightarrow$ projection



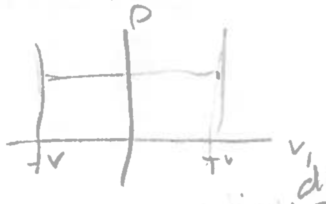
$v = \sqrt{\frac{2E}{m}}$

$= \sqrt{\frac{2kT}{m}}$

more dimensions? more components?

3d $\frac{1}{4\pi v} v^2 \sin\phi \, d\phi \rightarrow \quad dx$

$\longrightarrow$



$v_1^2 + v_2^2 + v_3^2 = U$

$\propto (\sin\phi)^{dN-2} d\phi \rightarrow \left(1 - \left(\frac{x}{v}\right)^2\right)^{\frac{dN-2}{2}} dx$

$dN \rightarrow \infty$

higher total # particles / dimensions

$\propto \exp\left(-\frac{\frac{1}{2} m v_1^2}{kT}\right)$

Boltzmann factor $\longrightarrow$ $\exp\left(-\frac{E_1}{kT}\right)$

example 2-state atom $0, E_1$

E_1 i.e 0,2eV

given T, what is the probability of finding it in each state?

=6000K

=600K

B.F. $\exp(0)$, $\exp(-0.387) = 0.679$

=1

$\exp(-3.87) = 0.021$

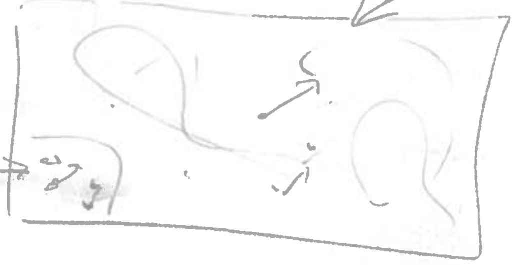
> Average energy in the atom? . . . .

From one to many particles

(A)

system of interest

heat bath



We need to normalise the probabilities
$p(\text{state } \alpha) = C \exp(-\beta E(\alpha))$

$$1 = \sum_\alpha p(\text{state } \alpha) = C \sum_\alpha \exp(-\beta E(\alpha)).$$
$$= C\, Z$$
$$p(\text{state } \alpha) = \frac{1}{Z} \exp(-\beta E(\alpha))$$

Z is very important. (i.e. heat capacity exercise)

It has a function similar to the phase space volume
Says something about available state space

$S = kT \ln \text{vol.}$ microcanonical (NVE)

$F = -kT \ln Z$ canonical (NVT)

Some more about averages.

$$\langle E \rangle = \frac{1}{Z} \sum_\alpha E(\alpha) \exp(-\beta E(\alpha))$$

$\langle A \rangle$ A

check:

$$\langle E \rangle = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{\partial}{\partial \beta} \ln Z$$

similar expressions for $\langle E^2 \rangle$