

Penrose scarf: a modular knit-in-one P3 (rhombic) Penrose tiling by Astrid S. de Wijn



This is a modular system for knitting a P3 Penrose tiling with no sewing. It can be used to make any flat shape approximately. For more information on Penrose tilings and why they are interesting, see the Wikipedia article on Penrose tilings.

This pattern includes charts for a scarf with two different length to width ratios as well as for smaller roundish objects: two different irregular hexagons that could be used as coasters and two different regular decagons. The long scarf in my gauge is 22cm wide and 156cm long. The coasters are 11cm \times 11cm and 8cm \times 15cm, and the decagons are 20 cm in diameter. You can of course also change the size by adjusting your gauge.

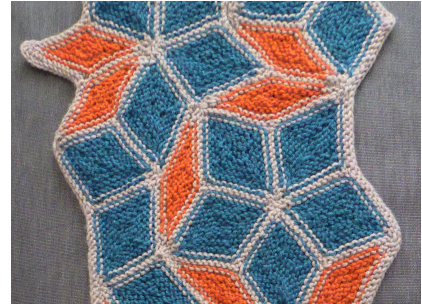
This is a fairly technical pattern. I recommend practicing by doing some of the smaller objects first. For the decagons, I have written out a very detailed pattern and description. In a much larger gauge, they can be used as nearly round cushion covers. Doing the decagons with the detailed pattern should give you a chance to try out all the techniques you need and get some experience with reading the charts.

If you find any mistakes, or if you find something hard to understand, please let me know. If you sell items based on this pattern, please include both the pattern and credit to me. If you create and distribute another pattern based on this technique, please also include a credit to me. If you'd like, you are welcome to send me cash anyway through paypal, or donate to the Electronic Frontier Foundation, the Free Software Foundation, or effective charities.



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The tiling system works with provisional cast-on (never a normal cast-on) and finishes with binding off at the edge. It has a reasonably ok-looking wrong side (see picture on the next page). In a larger tiling, it will often be necessary to kitchener two tiles together here and there. There are also a lot of ends to darn in.



1 Materials

- Yarn. Three colours with the same gauge. It is important that the Contrasting Colour (CC) and Main Colour (MC) for the fat rhombs tend to keep their shape. My usage for the long version, and estimate for the short version between ():
 - Borders (CC): Knit at home superfine merino wool, 100% merino, 105m/50g, 21-22 stitches/10cm. 126g = 265m (105 = 220m) colour 307, sand.
 - Fat rhombs (MC1): Knit at home classic wool, 100% superwash wool, 100m/50g, 21-22 stitches/10cm. 138g = 276m (109g = 219m) colour 433, bluegreen.
 - Skinny rhombs (MC2): Du store alpaka sterk, 40% alpaca 40% merino 20% nylon, 137m/50g, 22 stitches/10cm. 38g = 104m (31g = 85m) colour 842, burnt orange.
- Double-pointed needles that will give you the gauge you want. I used 3.5 for a gauge of about 21 stitches/10cm.
- Material for provisional cast-on: some smooth scrap yarn and depending on your method maybe a crochet hook.
- It is convenient to have a thinner double-pointed needle for transferring live stitches from the second side of a provisional cast-on.
- A tapestry needle for darning in the ends.
- It can save you a lot of darning in if you know how much yarn 24 stitches takes. We'll call this length l . For my gauge this was about 40 cm.
- A large amount of scrap yarn to use for holding stitches on.
- Two small stitch holders. They'll need to hold at most 4 stitches.
- For safety, you can use some thinner thread as a lifeline.

2 Abbreviations and glossary

RS	right side	byf	bring yarn forward
WS	wrong side	m1	make one: with the left needle lift the yarn between the needles from front to back, and knit into the back of this loop*
MC1	main colour for fat rhombs	kfb	knit once normally into the front of the loop, and once into the back of the loop
MC1	main colour for skinny rhombs	wt	wrap and turn [†] : s1 byf, slip the stitch back onto the right needle, turn the work and continue with the next row.
CC	contrasting colour for edges		
k	knit		
k2tog	knit the next two stitches together		
p2tog	purl the next two stitches together		
s	slip stitch as if to purl		

*You can also use a crochet hook to lift the yarn.

[†]The use of wrap-and-turn for short rows here is done so that the WS will look nice and symmetric. I have not tested if this will

Notes on provisional cast-on

All provisional casting on is done with the RS of the work facing you.

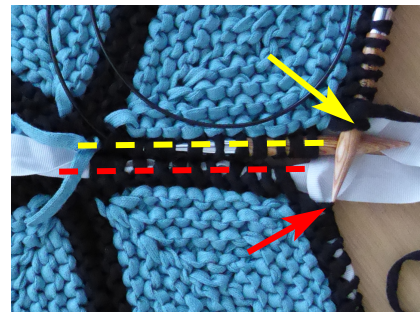
By provisional cast-on I mean that you should end up with a single strand of working yarn equivalent to one row of stitches. You can do this, for example, by picking up stitches through a crocheted chain. This will be the first row of the rhomb, which is why afterwards you continue with row 2.

When you are ready to knit another rhomb in the other direction, you have to unravel the provisional cast-on and transfer the live stitches to a needle. You may then end up with one extra stitch. This is corrected by k2tog on the next row (details below).

Notes on kitchener

In order to make the vertex of the rhomb edges look nice, you will need to use a modified setup for the kitchener grafting.

Normal garter stitch kitchener starts with sticking the yarn purlwise through the first stitch on the front needle, and then purlwise through the first stitch on the back needle. The normal repeat starts. Instead of this, you should also involve the "zeroth stitches" for both needles: the stitches not on the needles, but right next to them on the same rhombs. The zeroth stitch on the back needle is the one that you just been knitted with the yarn tail you are using for the kitchenering. In the picture on the right, the front needle is indicated by the dashed red line, and the back needle by the dashed yellow line. The corresponding arrows point to the corresponding zeroth stitches.



3 Tiling

The rhombs are knitted one by one following the charts which is included at the end of this pattern. The charts shows how the rhombs are connected, and if they are fat or skinny. Each rhomb in the chart is marked with a number indicating the order that the rhombs should be knitted in. The arrow in the rhomb indicates the direction of the knitting. The tip points towards the blunt corner in the middle of the final row. The non-pointed end of the arrow is at the blunt corner in the middle of the first row. The small lines across the sides at the corner indicate that you should not cut the CC yarn immediately, but at the very least leave a tail long enough to complete the future rhombs connected in this way. The detailed instructions for the two different types of rhombs are in the next section.

For each rhomb that you knit, including the very first one, go through the following steps one by one:

- 1 Determine what type of rhomb this is going to be, fat or skinny.
- 2 Determine where, if anywhere, the first row of the rhomb is attached to the previously existing work. Place any live stitches you will not be needing onto some scrap yarn. Put the live stitches you will be needing onto your needles. If necessary, unravel a provisional cast on. Whenever you do this, you may end up with an extra stitch, 13 instead of 12.

work for other methods of doing short rows.

- 3 If there is a small line on the right side of the rhomb connecting it to a previously knitted rhomb, you should have some CC dangling from the work in the right place. You can continue with that. Otherwise, take a CC skein that is not attached to the work.
- 4 With the arrow pointing upward, you should start at the right sharp point of the new rhomb and with the RS facing you.
- 5 You may still be missing some stitches. There are four possible situations:
 - a You have all the 24 stitches you need (or 25, or 26). Knit all 24 stitches for the first row of the rhomb (or k2tog once or twice in the middle if necessary to end up with 24 stitches).
 - b You have the first 12 (or 13) stitches, but not the second 12 (or 13). Knit the 12 stitches first (or k11 k2tog), then provisionally cast on 12 new stitches.
 - c You do not have the first 12 stitches, but you have the second 12 (or 13). Provisionally cast on 12 new stitches, then knit the 12 existing stitches (or k2tog k11).
 - d You may not have any of the stitches, for example for the first rhomb. Provisionally cast on 24 stitches.
- 6 Turn the work and knit according to the instructions on the next page for the correct rhomb, starting from the second row (WS), and finishing with the second to last row (WS).
- 7 Determine if the final row of the rhomb needs to be attached to any of the existing work. If it does not need to be attached, just knit the final row of the rhomb as written in the instructions. If it needs to be attached, it should only be at the upper right of the new rhomb, and will be indicated in the chart with a zig-zag line along the edge. In this case, knit the final row of the rhomb as written in the instructions, except for the last 12 stitches. Cut the yarn, leaving a length $\frac{1}{2}l$ plus a little bit. Put the 12 stitches from the existing work that you need to attach to on a needle. Kitchener the two sets of 12 stitches together using the end you've just cut. Include the zeroth stitches as described above.
- 8 Determine whether or not to cut the CC yarn and cut it only if necessary. If there is a little line on the right side of the rhomb in the diagram connecting it to a rhomb that you have not yet knitted, you should not cut the CC yarn. You will need it later for that future rhomb, and not cutting it now will save you darning in two ends. If you do not have enough separate skeins, you can count how many more rhombs you need to knit with this skein, and cut the yarn at the appropriate length (i.e. $4l$ times the number of rhombs left).
- 9 Place the live stitches on scrap yarn 12 at a time for easy retrieval later, except of course if you need them immediately for the next rhomb.

4 Instructions and diagrams for the rhombs

On the next two pages, there are the instructions and diagrams for the rhombs. In the instructions, the number between [] indicates how many stitches before the end the row ends, and therefore also where the next one begins. The diagrams are just a visual representation of the instructions, for those who prefer to knit that way. They are not in a standard diagram style that I'm aware of, because I could not find a way to indicate wt and keep track of the number of stitches you turn before the end in a convenient way.

Fat rhomb

Start with CC.

1. k24 [0] (RS)
2. k11 k2tog k10 [1] (WS, 23 stitches)

Switch to MC, start where you left off, 1 stitch from the end.

3. s1 k9 k2tog k8 wt [2] (22 stitches) [‡]
4. k17 wt[§] [3]
5. k7 k2tog k7 wt [3] (21 stitches)
6. k6 k2tog k6 wt [4] (20 stitches)
7. k11 wt [5]
8. k4 k2tog k4 wt [5] (19 stitches)
9. k4 k2tog k2 wt [6] (18 stitches)
10. k5 wt [7]
11. k3 wt [8]
12. k2 wt [8]
13. k3 wt [7]
14. k1 kfb k3 wt [6] (19 stitches)
15. k3 kfb k4 wt [5] (20 stitches) [¶]
16. k10 wt [5]
17. k5 kfb k5 wt [4] (21 stitches)
18. k6 kfb k7 wt [3] (22 stitches)
19. k16 wt[§] [3]
20. k7 kfb k9 s1 byf[¶] s1 [0] (23 stitches)

Cut the MC yarn, leaving about 8 cm for darning in. Switch to CC and start from the beginning. Make sure that the CC is wrapped around the

last stitch like a wt.

21. s1 k10 kfb k10 wt (24 stitches) [1]
22. k23 [0]

Skinny rhomb

Start with CC.

1. k24 [0] (RS)
2. k23 [1] (WS)

Switch to MC.

3. s2 k8 k2tog k8 wt [3] (23 stitches) ^{**}
4. k16 wt [4]
5. k13 wt [6]
6. k4 k2tog k4 wt [7] (22 stitches)
7. k6 wt [9]
8. k3 wt [10]
9. k2 wt [10]
10. k3 wt [9]
11. k2 kfb k3 wt [7] (23 stitches)
12. k10 wt [6]
13. k13 wt [4]
14. k7 kfb k8 s1 byf[¶] s2 [0] (24 stitches)

Cut the MC yarn, leaving about 8 cm for darning in. Switch to CC and start from the beginning. Make sure that the CC is wrapped around the last stitch like a wt.

15. s1 k22 wt [1]
16. k23 [0]

[‡]While knitting the 3rd row, you can place the first two stitches (CC stitches) on a stitch marker and leave it dangling. This will prevent them from loosening, which will mess up their tension even more. You will have to put them back on row 19.

[§]This wt is a stitch too early. It should really be k1 wt k1 instead of just wt. However, that does not look good on the WS of the work, and this seems to be the only way to make it look ok.

[¶]From the next row onward, it may be convenient to split the stitches onto two needles, to prevent too much pulling on the stitches at the ends.

[¶]This will set up a simulated wt at this point when you darn in the ends, so that everything is nice and symmetric on the WS.

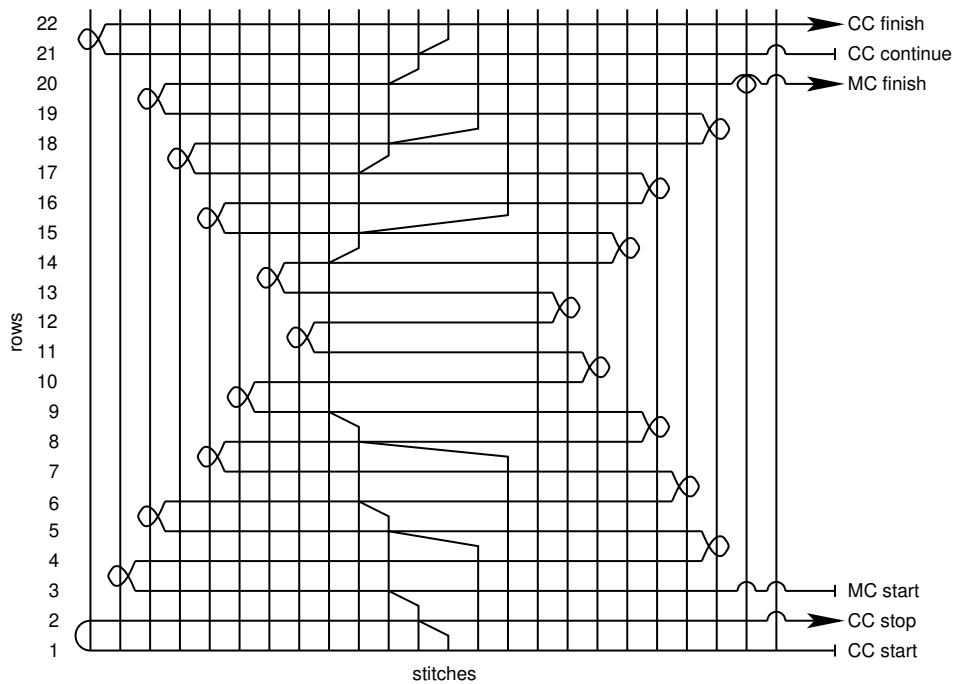
^{**}While knitting the 3rd row, you can place the first three stitches (CC stitches) on a stitch marker and leave it dangling. This will prevent them from loosening, which will mess up their tension even more. You will later have to put them back on row 14.

Diagrams

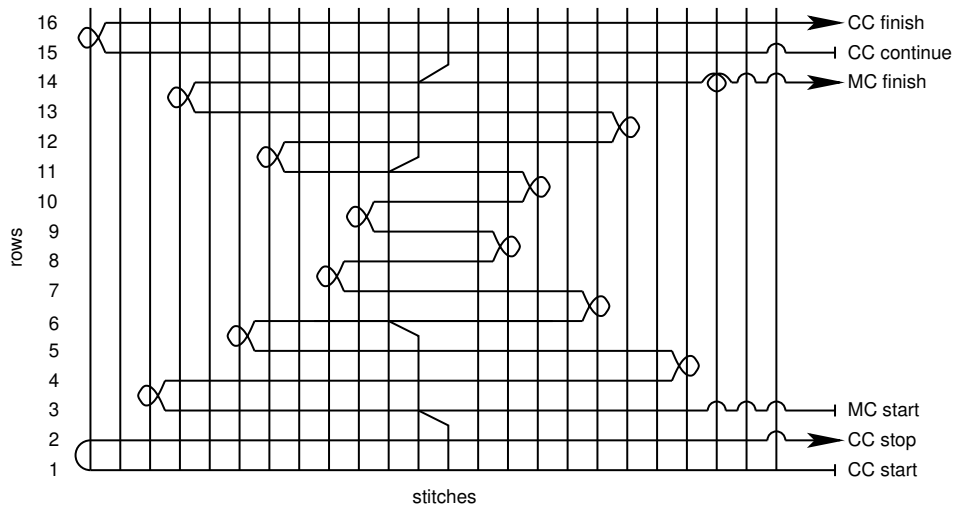
The horizontal lines indicate the working yarn. Start at the bottom right and follow the mostly horizontal lines. A line from right to left is a right-side row, from left to right is a wrong-side row. The vertical lines indicate stitch chains. All stitches are knit stitches.

⊥	stitch (k) or cast-on stitch	⌵	knit two stitches together (k2tog)
+	stitch (k)	⌶	knit in front and back of loop (kfb)
⤿	turn work	⌷ or ⌸	wrap and turn (wt)
↶	slip (s)	⌹	slip one, bring yarn forward (s1 byf)

fat rhomb



skinny rhomb



5 Edge and finishing

5.1 Darning in MC end and CC ends not at the edge

Darn in the loose ends of the MC, making sure to connect them in such a way as to complete the wrong-side pattern with the wt. The ends are already prepped for this and just need to be connected. Each end goes into the hole that the other end sticks out of. The result is shown in picture on the right.

You can also already darn in the loose ends of the CC that are not near the edge. In many cases, the CC ends will be at a hole where several corners of rhombs meet. Before darning them in, use these CC ends to close these holes, by threading them through the stitches and pulling it closed.



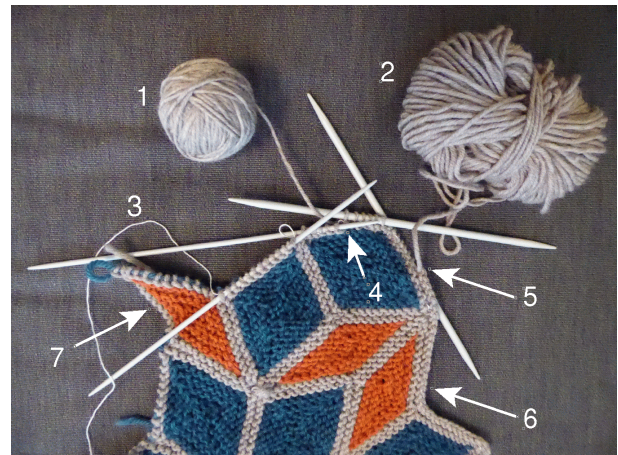
5.2 Edge

You will now go around the entire edge knitting two rounds simultaneously, with two skeins of CC. The picture on the right shows how knitting the edge will look as you are going along.

The edge is divided into edges of rhombs, each containing 12 or 13 stitches, depending on if they are the second side of a provisional cast-on or not. To start, pick a set of 12 or 13 stitches from some edge of a rhomb and transfer the first four stitches to a stitch holder, and the rest to a needle. Start here with edge round 1, which consists of just knitting, with the occasional increase and decrease at corners. The corners are treated differently depending on how sharp they are. To determine the sharpness, add up points as follows: 1 for each sharp corner of a skinny rhomb, 2 for each sharp corner of a fat rhomb, 3 for each blunt corner of a fat rhomb, and 4 for each blunt corner of a skinny rhomb. The table below gives recommendations how to handle each case. The angle

is actually 36° times the number of points. If you are perfectionistic, you may want to use a lifeline for some tricky corners, so that it's easy to go back and adjust your increases/decreases if you are not happy.

As you go along with round one, you will have to keep putting the next live stitches onto needles and will eventually run out of needles. Once you have made some progress or have used up almost all of your double-pointed needles, you should start edge round 2 by going back to the beginning. Move the first four stitches from edge round 1 to another stitch holder, then start round 2 from here. The second round consists only of binding off purl-wise, with occasionally p2tog rather than normal purl stitches (noted in the table below). Continue alternating between edge rounds 1 and 2.



1: the yarn for the first edge round. 2: the yarn for the second edge round. 3: lifeline. 4: The current progress of the first edge round. 5: the current progress of the second edge round (binding off). 6: finished edge. 7: live stitches from rhombs held on scrap yarn.

Finish each of these rounds where you started them and darn in the ends. The staggered construction of the two edge rounds should make the start and finish look more smooth. You can loop the loose end of the last stitch of the second round around the first stitch of the second round, to make a complete purl bind-off.

case	instructions	
1	It's a sharp point of a rhomb. One of the two edges at the corner will have 13 stitches and the other 12. Increase using m1 one stitch away from the corner in the 12-stitch edge, and 2 stitches away from the corner in the 13-stitch edge. There will be 3 stitches knitted in-between the two increases.	
2	If it is a sharp point of a fat rhomb	Do as 1.
	If it consists of two sharp points of skinny rhombs.	This does not happen in the scarf charts, but may happen in variations. I have not tested it, but you can probably treat this in the same way as if it was a single sharp corner. If you do not have a side with 13 stitches, then simply m1 at the corner and treat it as the 13th stitch of one of the two edges.
3	If it is a blunt corner of a fat rhomb.	k to the corner, kfb ⁺⁺ . Continue knitting.
	If it consists of the sharp corners of one fat rhomb and one skinny rhomb.	k to the corner, m1, continue knitting.
4	k to one stitch before the corner, m1, continue knitting.	
5	No increase or decrease needed.	
6	k2tog at the corner (one stitch from each edge).	
7	k2tog at the corner (one stitch from each edge).	
8	Knit to 2 stitches before the corner, k2tog, m1, k2tog. On the second edge round, p2tog at the corner. You can use a lifeline and test a few things out to make sure it looks ok.	
9	Untested. You should probably decrease at least 3 stitches. Use a lifeline and try a few things.	

5.3 Darning in CC ends

Darn in the remaining CC ends. There may be some holes left, especially where sharp points meet. Take a bit of CC yarn and thread it through the stitches around the hole, then pull it closed. Darn in the two new ends.

6 Possible modifications

I'm happy to help out with any modifications if you have a specific request, but here are some ideas and suggestions.

Other tilings

You can construct more charts yourself based on the tiling rules. Obvious ideas would be a triangular shawl and an afghan. There is an example rotation-symmetric Penrose P3 tiling on the Wikipedia page that could

⁺⁺ It is important to kfb here instead of m1, due to the construction of the fat rhomb. If you do not do this, there will be an MC loop pulled out in an awkward way.

be used as a basis for both. There are also many tiling generators you can find online. When deciding what order to knit the rhombs in, you should make sure that any kitchenering should always occur on the upper right corner of the second rhomb involved in the connection. In a properly constructed Penrose tiling, it should never be necessary to kitchener at the upper left corner of the newer rhomb. If it is the upper left, then due to the tiling rules it will be the upper right corner of the rhomb you are kitchenering the new rhomb to, and you could have done them in reverse. If it somehow occurs anyway that you have to kitchener the upper left, then you can finish the new rhomb according to the instructions, and instead undo the last 12 stitches of the older rhomb you need to attach it to and use that end for kitchenering. Also, you might want to avoid kitchenering directly to the second edge of a provisional cast-on, because then you may have one stitch too many. This can probably be fixed by treating the first two stitches at the blunt corner of the rhomb with 13 stitches as one, but I have not tested if this looks good or not.

Modifying the tiles

Besides modifying the charts, you could also modify the actual rhomb modules. It should be fairly simple to construct rhombs with the same shape but more stitches. If you want to try this on your own, take a look at the diagrams, because it should be possible to understand the construction from those. They show how often you have to increase/decrease and how the short rows become shorter. In the skinny rhomb, you decrease/increase on every 3rd row, while on the fat rhomb you decrease/increase on every 2 out of 3 rows. The wt is 3 stitches before/after the last one on the skinny rhomb, while it alternates between 1 and 2 stitches before/after on the fat rhomb. Things might get a bit messy in the centre.

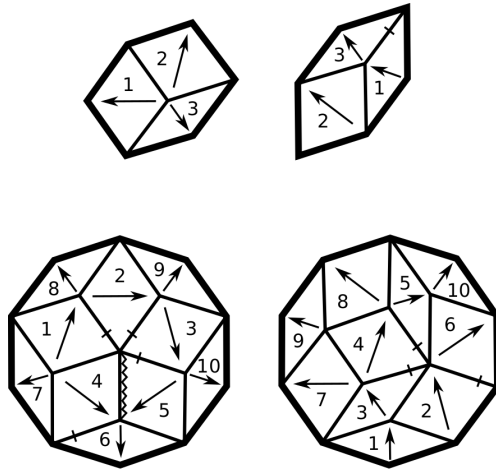
I don't recommend knitting the rhombs entirely in the MC, because it seems to lead to cosmetic problems in points where many sharp corners meet, where holes appear that need to be closed. It's much easier to do this nicely if all the corners are in the same colour yarn. If anyone figures out a trick to deal with that, please let me know. If you did manage to do this, the entire tiling can be done with just three colours, because it has been shown that any P3 Penrose tiling can be coloured with just three colours in such a way that no two tiles of the same colour share an edge^{**}.

^{**}Sibley TQ, Wagon S. 2000. Rhombic Penrose tilings can be 3-colored. *The American Mathematical Monthly* 107(3): 251-253.

7 Charts

For the shorter scarf, skip all grey rhombs.

hexagons (coasters)



decagons (pot holders or cushion covers)

scarf

