

bond stretching, bending, torsion etc

FK 7029

(3)

easiest if just full 3d eq of motion per particle, (no rigid constraints)

- treat all atoms individually
- take care to use small displacement for tightly bound atoms \Rightarrow not so efficient.

rigid constraints

- do explicitly
- for large molecules become difficult, MD easier.

Important point: trial moves need not be physical (unlike MD)

Cluster moves.

as long as the reverse has same prob \Rightarrow detailed balance

Remember Ising model at low T, $\uparrow\uparrow\uparrow\uparrow\uparrow$ $\downarrow\downarrow\downarrow\downarrow\downarrow$

- in principle conf all \uparrow and all \downarrow are connected, by very unlikely sequence of moves due to high energy barrier

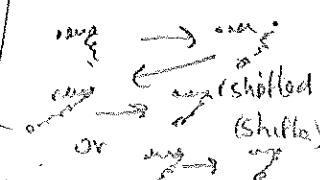
Flipping 1 spin at a time will not make this transition,

(common problem in MC & MD)

MC moves can bypass U-barriers: Cluster moves.

- internal interactions in cluster unchanged
- interaction with environment changes.

example: moving C.O.M of molecule



\Rightarrow best clusters: $\Delta E \approx 0 \Rightarrow$ high acceptance.

or tune acceptance probs

example: Ising model.

$\uparrow\downarrow\downarrow\downarrow$ etc. $H = - \sum_{nn} J S_i S_j$

\uparrow assume positive.

$U = (N_a - N_p) J$
 # anti-parallel # parallel

cluster: connecting neighbours

- not if antiparallel,
- prob p if parallel

suppose we flip the subset of cluster, which

will send $N_p' = N_p + \Delta$
 $N_a' = N_a - \Delta$

$U' = U - 2J\Delta$

prob of forward move generated: $p^{n_c} (1-p)^{N_p - n_c}$
 backward move: $p^{n_c'} (1-p)^{N_p' - n_c'}$

connected spins $N_p - n_c$ broken spins

$P(\text{forward})/P(\text{backward}) = (1-p)^{-\Delta}$

violate detailed balance? $n_c' = n_c$ must flip some cluster back

detailed balance

original state, $P(\text{forward})$ $\xrightarrow{\text{generate}}$ $P(\text{accept forward})$
 new state, $P(\text{backward})$ $\xrightarrow{\text{generate}}$ $P(\text{accept backward})$

Suppose \Rightarrow (31)
 (So not $\exp(-\beta \Delta U)$ can't be, because $P(\text{forward}) \neq P(\text{backward})$)

$\exp(-\beta U) \quad \exp(-\beta(U - 2J\Delta))$

$P(\text{forward}) / P(\text{backward}) = \exp(+\beta 2J\Delta) = (1-p)^{-1}$

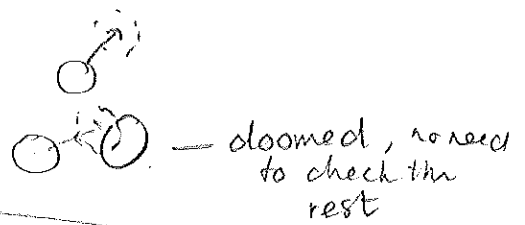
$\Rightarrow \exp(-\beta 2J) = 1-p$

\Rightarrow always accept and you speed up convergence!

In general not always possible

Early rejection: figure out early on that a move is doomed and never calculate all of it.

- hard-core interactions
- strongly repulsive component.
- if $\Delta U >$ something



Ensembles in MC ; same as thermostats in MD; get right dist

- So far, done NVT

Microcanonical MC

- generate trial move as usual
- extra variable: E_D
- modify acceptance,
 - if $\Delta U < 0$; accept, $E_D += \Delta U$
 - if $\Delta U > 0$, $E_D > \Delta U$; accept $E_D - = \Delta U$
 - , $E_D < \Delta U$; reject

acceptance not random, but generation of trial move is wiggle around conserved energy. $U + E_D$ conserved. in principle should include kinetic term as well not used much.

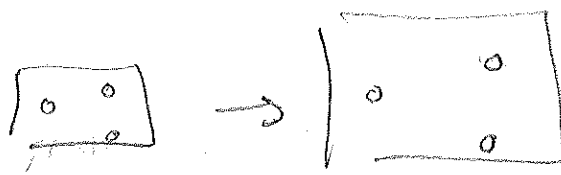
[Faint handwritten notes and diagrams at the bottom of the page, including some mathematical expressions and diagrams of particles.]

MC in NPT ensemble

- common experimental situation
- construction similar to barostat

rescale the system

V as variable



distribution

$$\int dr \exp[-\beta(U+PV)] \cong \int ds \underbrace{V_{\text{new}}^N}_{\text{Jacobian}} \exp[-\beta(U+PV)]$$

↑
scaled coordinate

what we need to construct

- step 1 trial move, either (a) normal NVT move or (b) change in volume: $r \rightarrow r \left(\frac{V_{\text{new}}}{V_{\text{old}}}\right)^{1/3}$

- step 2 accept (a) according to Metropolis or (b) according to

$$P = \min\left(1, \underbrace{\exp[-\beta(\Delta U + P\Delta V)]}_{\tilde{p}} \left(\frac{V_{\text{new}}}{V_{\text{old}}}\right)^N\right)$$

detailed balance

$$P(\rightarrow) \cdot V_{\text{old}}^N \exp[-\beta(U_{\text{old}} + PV)] = P(\leftarrow) \cdot V_{\text{new}}^N \exp[-\beta(U_{\text{new}} + PV)]$$

$$\min(1, \tilde{p}) = \min(1, \frac{1}{\tilde{p}}) \tilde{p}$$

true!

end lecture 10